

Components of a Synchrotron

- **The electron storage ring (accelerator physics)**
- **The experimental beamlines (users)**
 - Front-end
 - Optics
 - End-stations

Ideally, the design of an experiment starts at designing the storage ring

Emittance: ε

The brightness of the light depends on how *tightly the electron beam is squeezed*.

spatial deviation of the electron from the ideal orbit is σ_x , in the plane of the orbit with **angular spread** σ_x' .

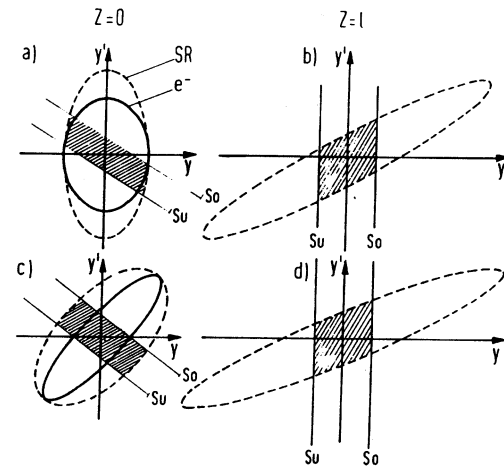
σ_y , vertical to the plane of the orbit, with σ_y' .

Emittance ε (nm-rad) is expressed as

$$\begin{aligned}\varepsilon_x &= \sigma_x \sigma_x' \\ \varepsilon_y &= \sigma_y \sigma_y'\end{aligned}$$

**Emittance is conserved
(Liouville theorem)**

**Typical emittance of 3rd generation
ring: 10 - 1 nm-rad**



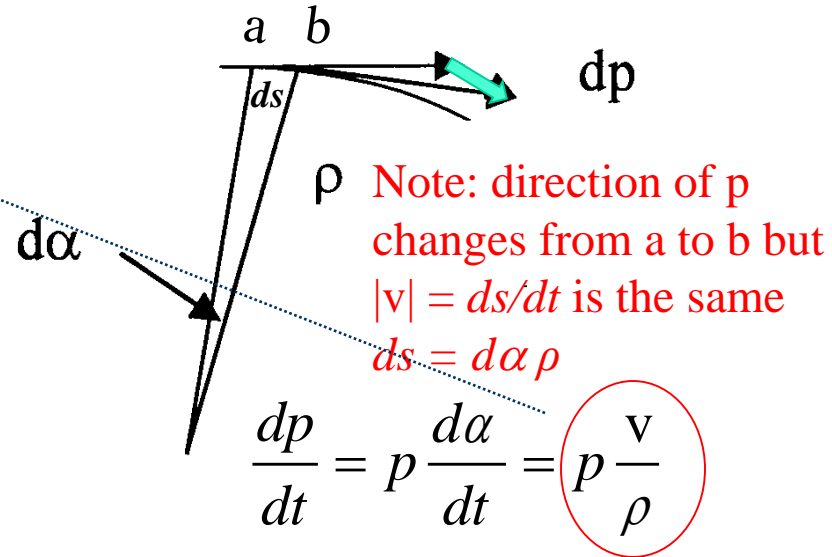
Phase space ellipsoids

Energy loss to synchrotron radiation

$$P = \frac{2}{3} \frac{e^2 \gamma^2}{m_0^2 c^3} \left| \frac{dp}{dt} \right|^2$$

$$\begin{aligned} v &\sim c, pc = E \\ \gamma &= E/m_0 c^2 \end{aligned}$$

$$P = \frac{2}{3} \frac{e^2 c}{(m_0 c^2)^4} \frac{E^4}{\rho^2}$$



$$\Delta E = \oint_{orbit} P dt = P \frac{2\pi\rho}{c} \longrightarrow \Delta E [keV] = 88.5 \frac{E^4 [GeV^4]}{\rho [m]}$$

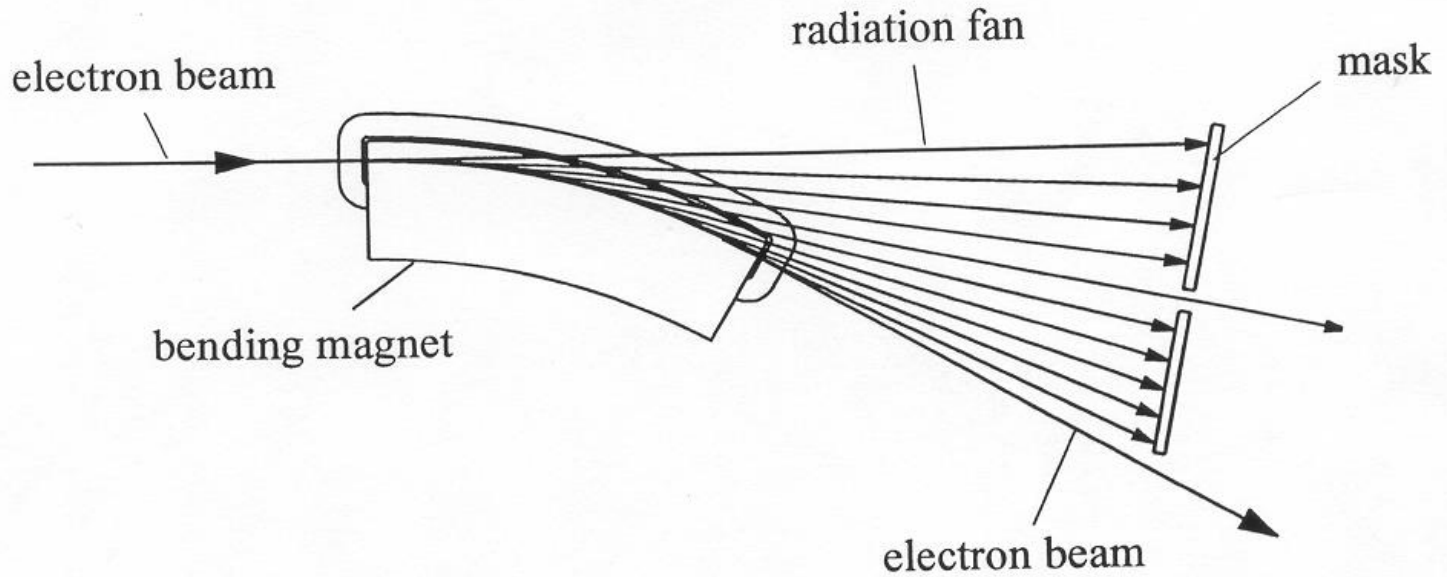
Energy loss per turn per electron \rightarrow synchrotron radiation !

For a 3.5 GeV ring with a radius of 12.2 m, the energy loss per turn is $\sim 10^6$ eV \Rightarrow Most storage rings built to-day are \sim GeV rings

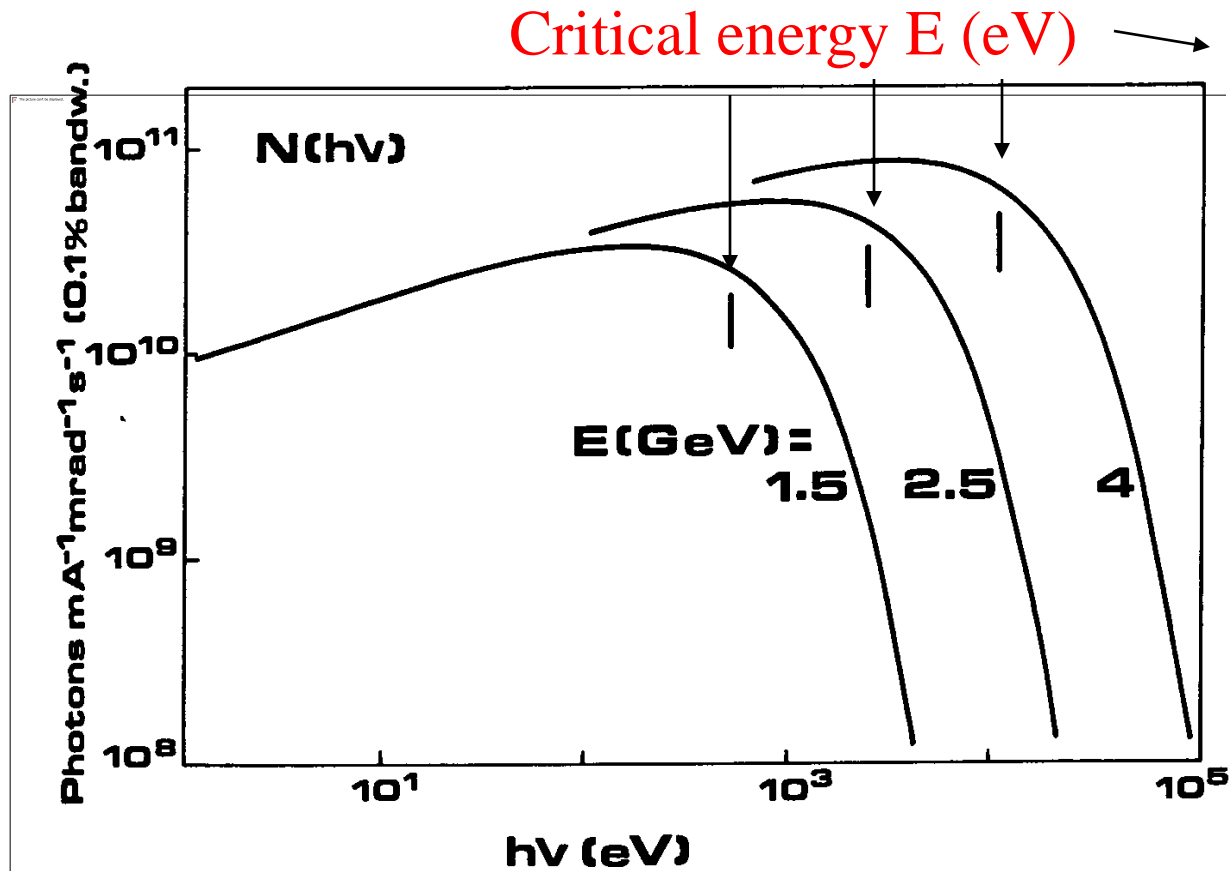
How is synchrotron radiation generated ?

- There are three ways
 - *To bend* the electron with a **bending magnet (dipole radiation)**
 - *To wiggle* the electron with a periodic magnetic field called **insertion device**
 - high field with a big bent: **wigglers**
 - medium field with many small bents: **undulator**

Synchrotron radiation from BM



BM (dipole) spectrum and critical energy



Critical energy E (eV)

half of the radiated power is into photons with energy above E_c , and half of the power is below

$$E(eV) = 12398.5 / \lambda(\text{\AA}) \quad \lambda_c = \frac{12386.5}{800(eV)} (\text{\AA}) = 15.5 \text{\AA}$$

critical wavelength

critical energy

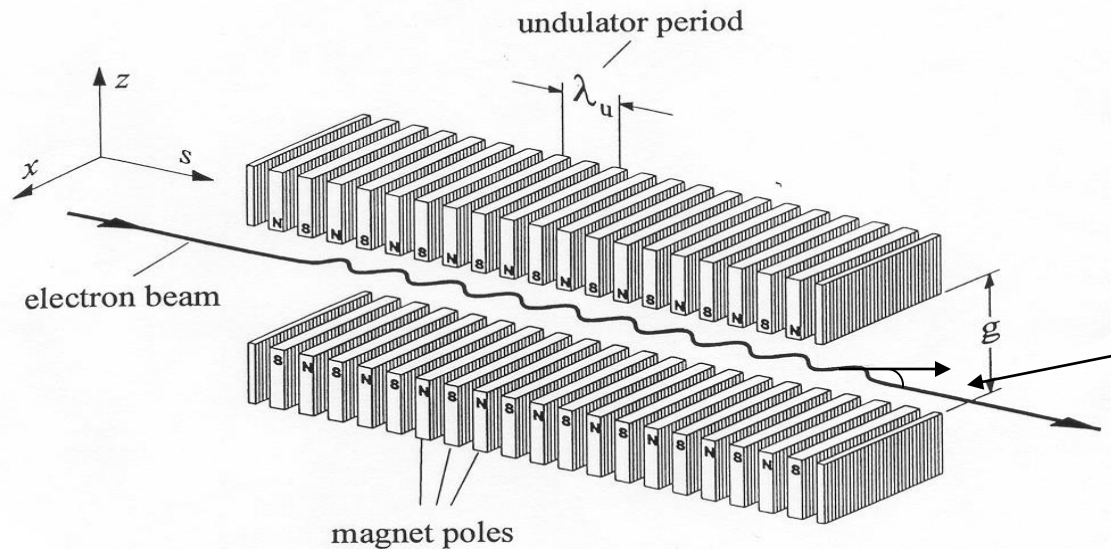
Wigglers and undulators

Alternating magnetic structures
short period, large bend: wiggler
long period, small bend: undulator

**strength
parameter**

$$K = 0.934 \lambda_u [\text{cm}] B_o [\text{T}]$$

peak magnetic field



angle of deflection
 $\delta = K / \gamma$

Undulator equation

$$\lambda_1(\Theta) = \frac{\lambda_u}{2\gamma^2} \left[1 + \frac{K^2}{2} + \gamma^2 \Theta^2 \right]$$

Θ : angle of observation

$K \gg 1$: large field, sizable bend, negligible interference \Rightarrow wiggler

$K < 1$: short period, modest bend, interference \Rightarrow undulator

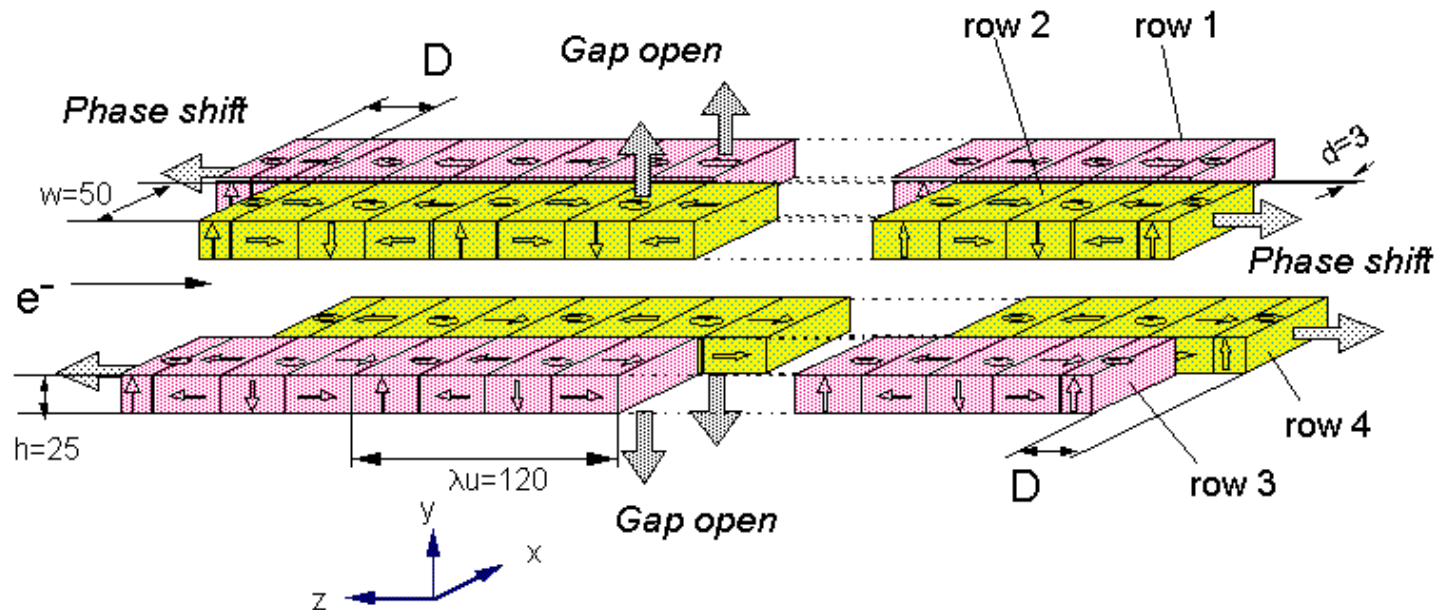
Fundamental energy/wavelength on axis

$$\varepsilon_1 [keV] = \frac{0.950 E^2 [GeV]}{(1 + K^2 / 2) \lambda_u [cm]}$$

$$\lambda_1 \left[\frac{\text{\AA}}{A} \right] = \frac{13.06 \lambda_u [cm] (1 + K^2 / 2)}{E^2 [GeV]}$$

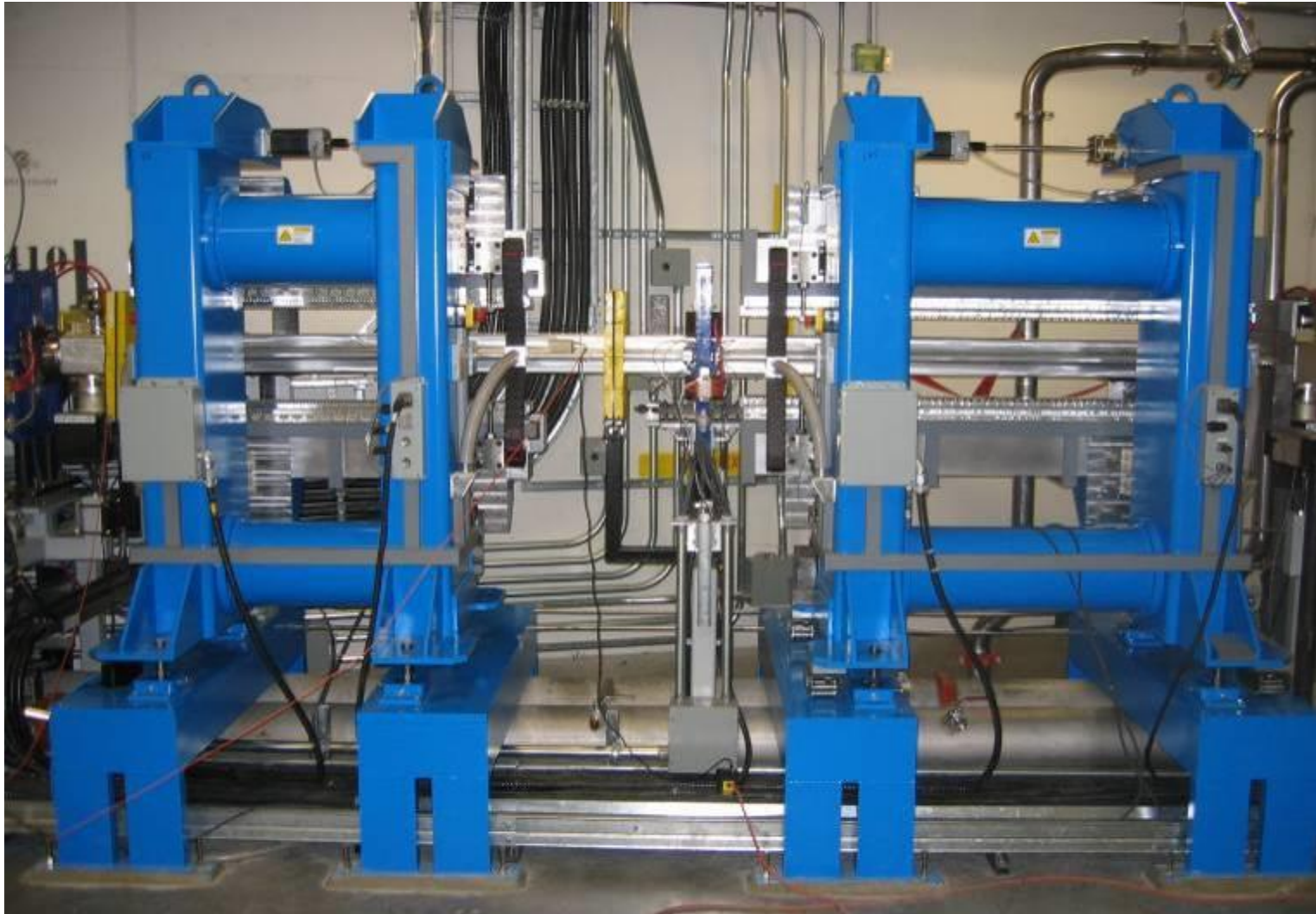
Modern Insertion Devices (APPLEII)

- Energy tunable by adjusting the gap
- In vacuum small gap undulator
- Polarization tunable by tuning the magnetic structures: EPU



The magnetic structure of the APPLE II consists of two pairs of arrays of permanent magnets.

Chicane Insertion Device at CLS

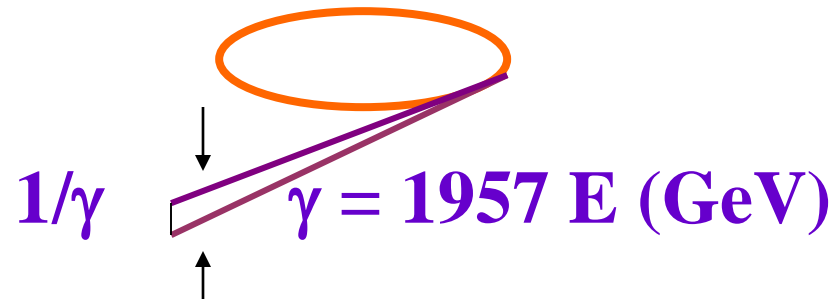


SGM

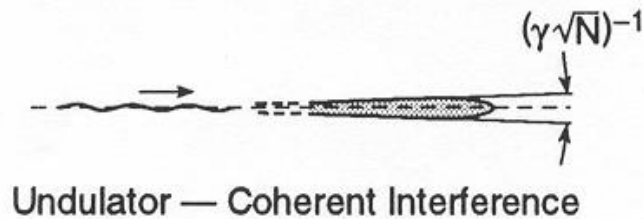
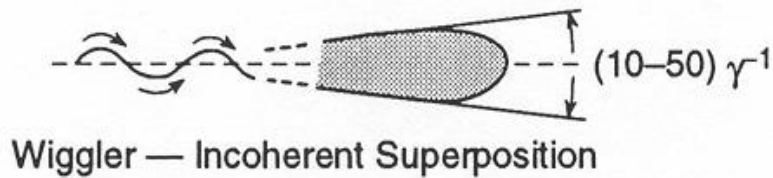
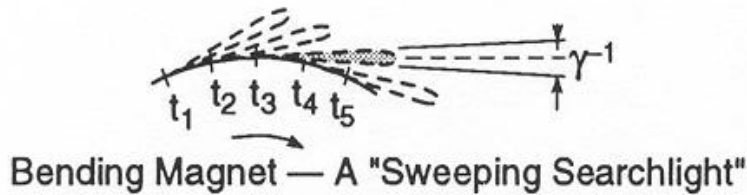
PGM

Characteristics of synchrotron radiation

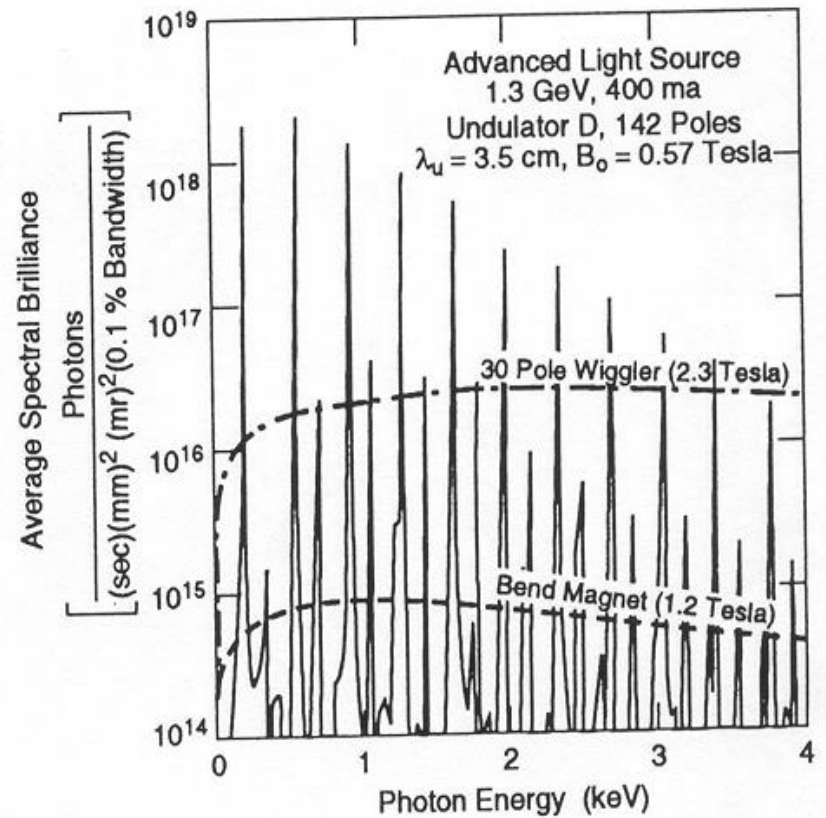
- **Spatially distribution:** highly collimated half angle $\psi = 1/\gamma$, $\gamma = 1975 \text{ E (GeV)}$.
 - bending magnet: $1/\gamma$
 - undulator: $1/(\gamma\sqrt{N})$
 - wiggler: $\gg 1/\gamma$



- **Spectral distribution:** continuous at BM and wiggler sources, spike like peaks in undulator source due to interference effects



Spatial distribution

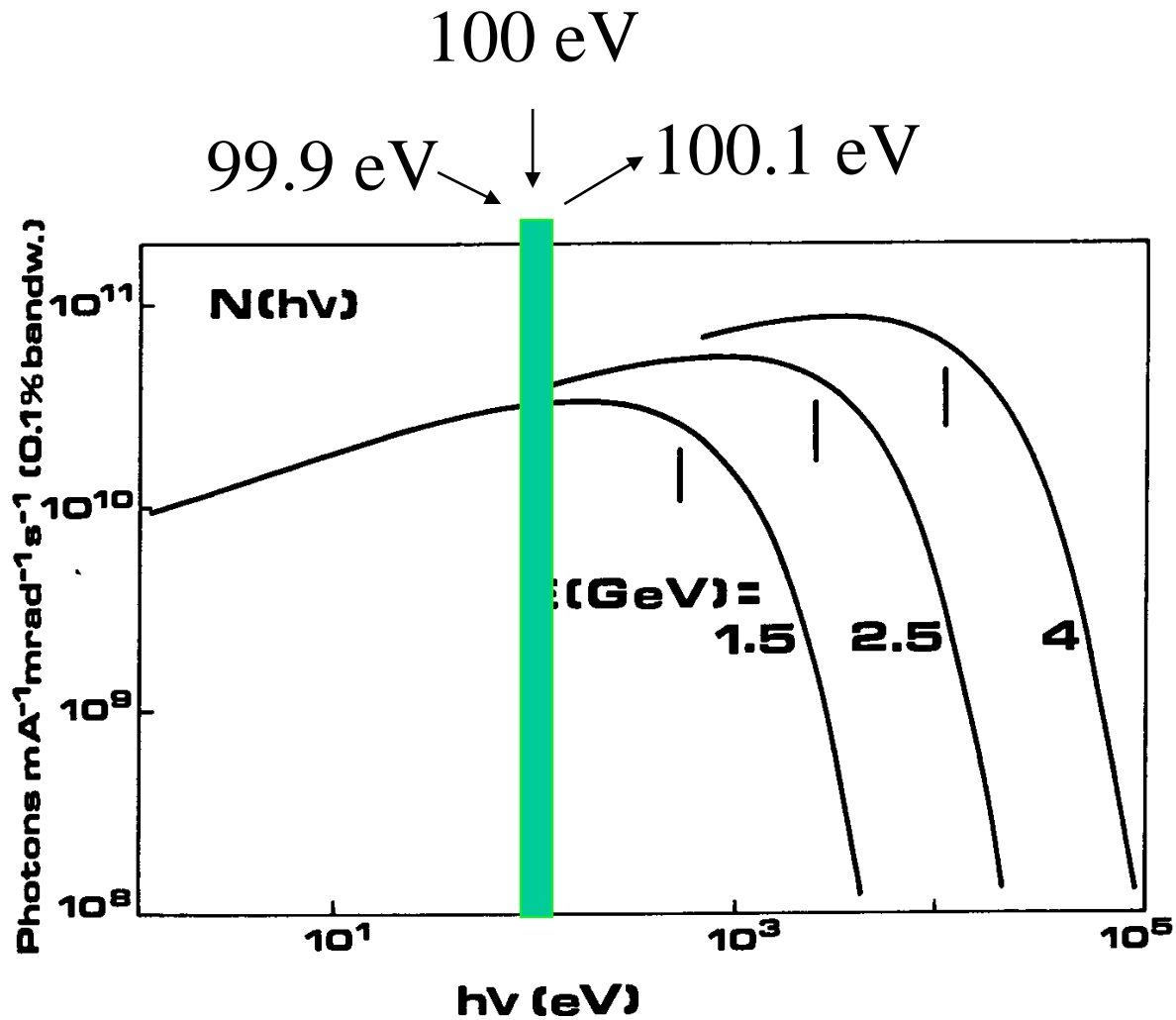


Spectral distribution

Why is synchrotron radiation special For materials analysis ?

- Tunability (IR to hard x-rays)
- Brightness (highly collimated)
- Polarization (linear, circular, tunable)
- Time structure (short pulse)
- Partial coherence (laser-like beam from ID)

0.1% band path



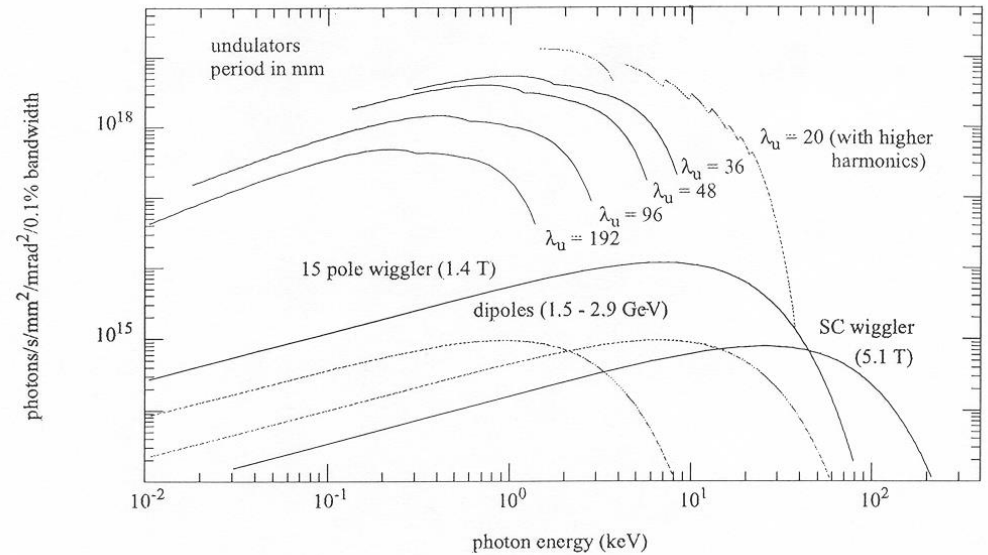
Brightness and flux

- **Brightness:** photons per 0.1% bandwidth $\text{sec}^{-1} \text{mm}^{-2} \text{mrad}^{-2} \text{mA}^{-1}$
- Number of photons per bandwidth, per unit time per unit source area, per unit solid angle (*inherent to ring design: emittance*)
- **Flux:** photons per 0.1% bandwidth $\text{sec}^{-1} \text{mA}^{-1}$
Number of photons per bandwidth per unit time (scaled to ring current and orbit of arc, *can be improved by beamline optics*)

Brightness and Flux revisited

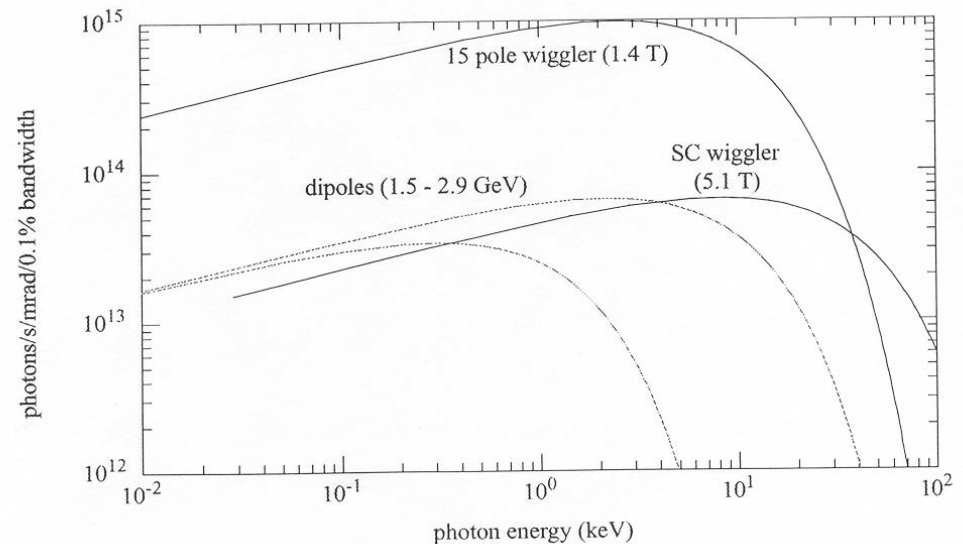
Brightness:

No. of photons per sec
per mm^2 (source size)
per mrad^2
(source divergence)
per mA, within a 01.%
band width



Flux:

No. of photons per sec
per mrad angle of arc
within a 01.%
band width



Spectral Distribution of a BM source

BM radiation has a wide energy distribution characterized by λ_c

$$\lambda_c (\text{\AA}) = \frac{18.6}{E^2 B} = \frac{5.6\rho}{E^3} = \frac{4\pi\rho}{3\gamma^3}$$

note: $\gamma = 1957E(\text{GeV})$

Example: SRC (WI)

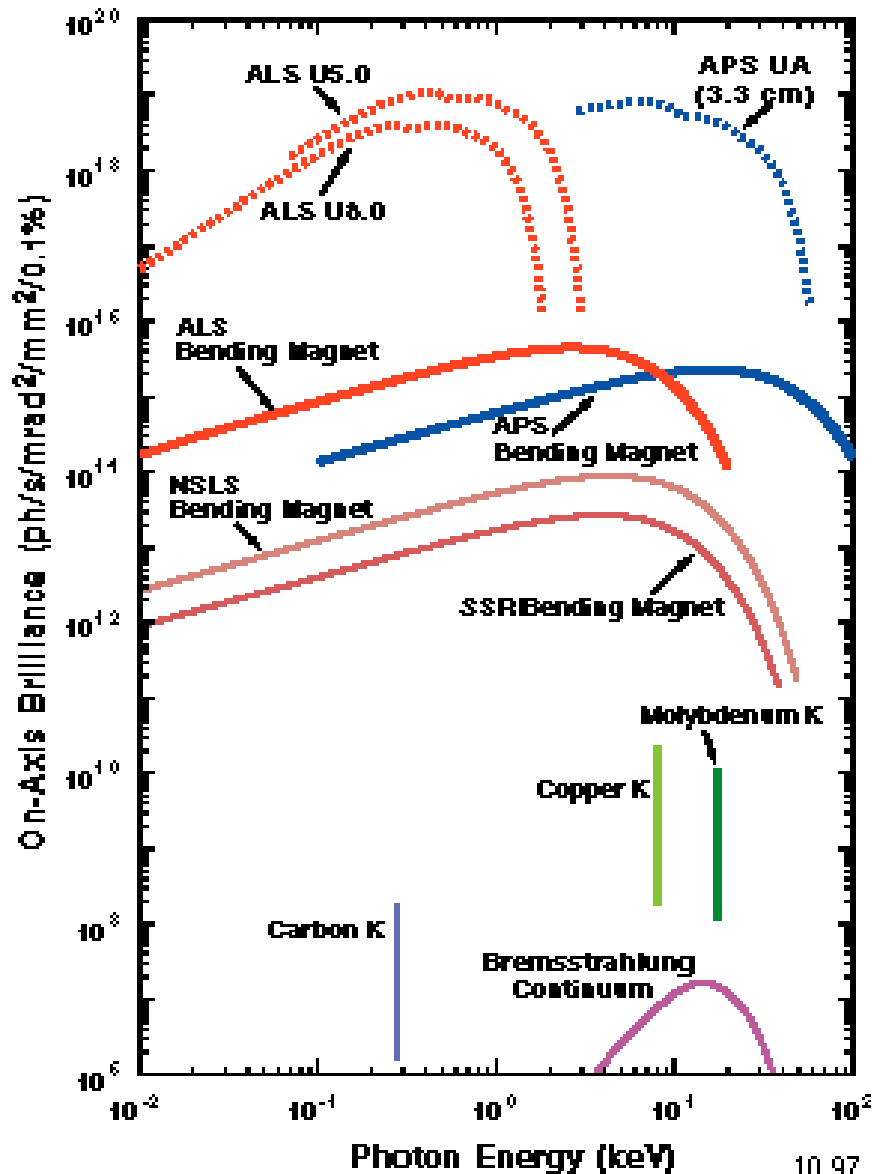
$E = 0.8 \text{ GeV}$,

$\rho = 2.0833 \text{ m}$,

$\lambda_c = 22.7 \text{ \AA}$

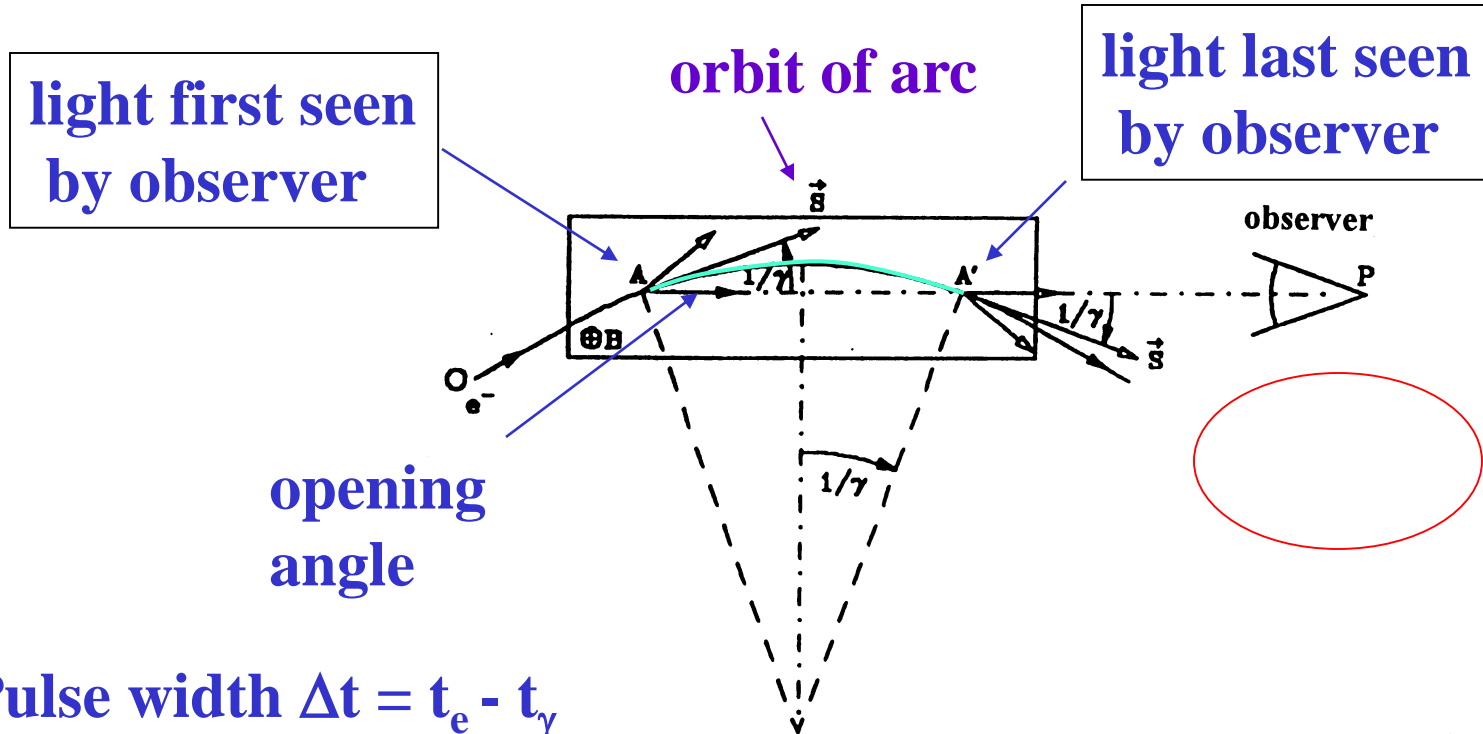
$E_c = 12398.5/22.7 = 546 \text{ eV}$

Exercise: Comment on the similarity between Bremsstrahlung and SR



Why is there a wide spread in photon energy?

Qualitative understanding: Uncertainty Principle



Pulse width $\Delta t = t_e - t_\gamma$
 photon travels faster than electron

$$\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta\gamma c} - \frac{2\rho \sin(1/\gamma)}{c} \approx \frac{4\rho}{3c\gamma^3}$$

$$\Delta E \cdot \Delta t \geq \hbar/2$$

$$\Delta E_{SR} \geq \hbar/2\Delta t \geq 3\hbar c\gamma^2/2\rho$$

typically $\sim \text{keV}$

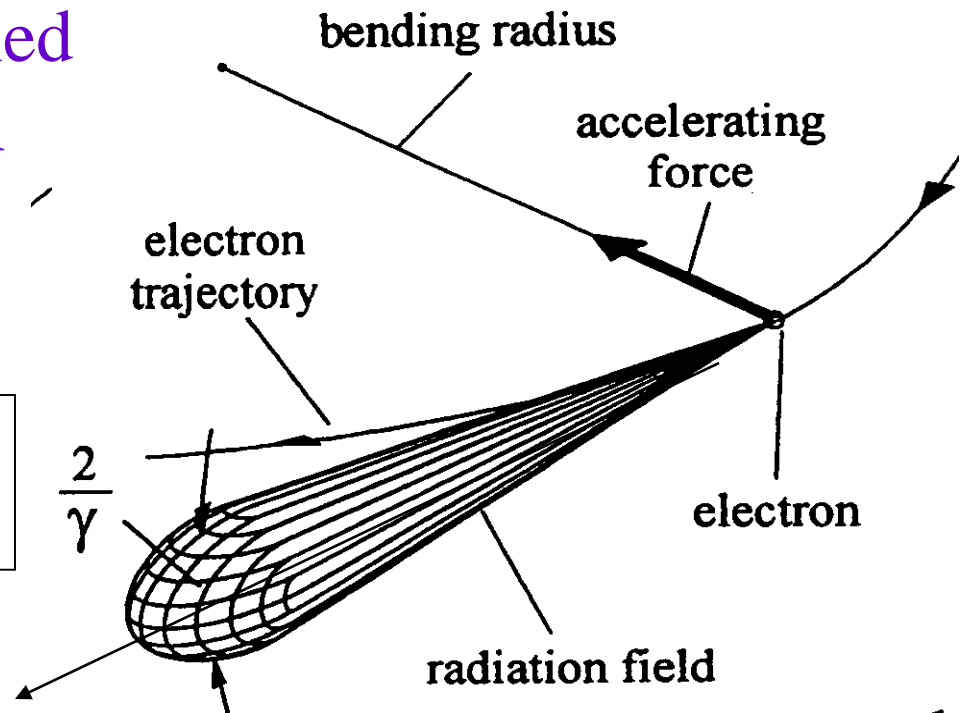
Exercise: Show this; hint: $\gamma \gg 1$; $\sin x = x - x^2/3! + \dots$

The opening angle: angular distribution

Opening angle, ψ , is defined as the half angle of the SR above and below the orbit plane.

$$\psi \sim (1/\gamma) 0.57 (\lambda/\lambda_C)^{0.43} \text{ (radian)}$$

$$\psi \sim (1/\gamma)$$



We recall that $\gamma = 1957 E(\text{GeV})$, hence high energy electrons will produce a more **collimated** photon beam.

Also the shorter the wavelength the more collimated the light.

e.g.: X-ray is more collimated than IR

Polarization

Degree of polarization

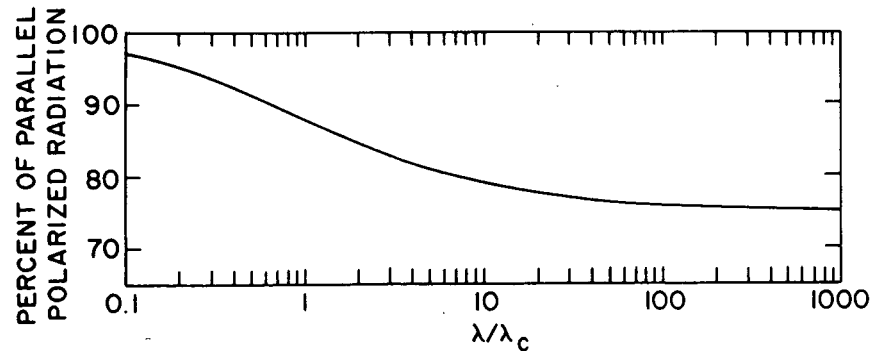
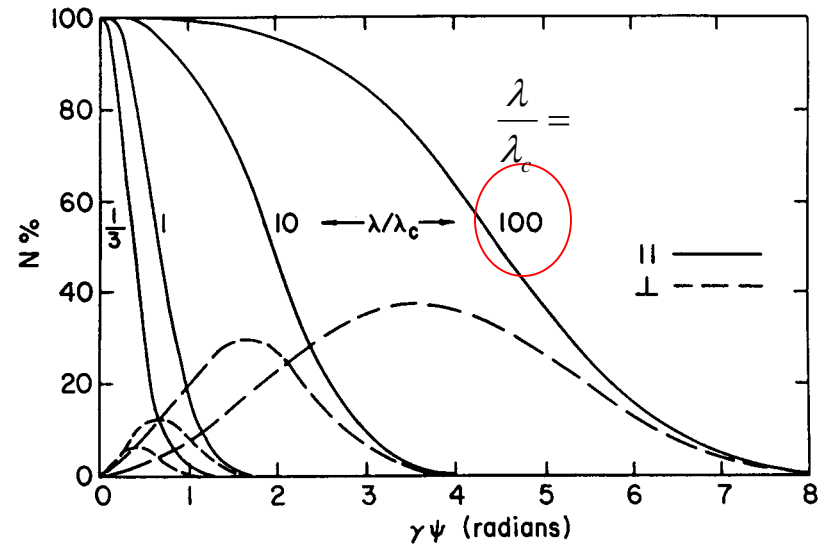
$$p = (I_{\parallel} - I_{\perp}) / (I_{\parallel} + I_{\perp})$$

$$\psi \sim (1/\gamma) 0.57 (\lambda/\lambda_C)^{0.43}$$

(radian)

$$\psi \gamma \sim 0.57 (\lambda/\lambda_C)^{0.43}$$

**Polarization increases
at shorter wavelength**



The radio-frequency cavity

Where is a cavity located in a storage ring?

At a straight section

What does it do?

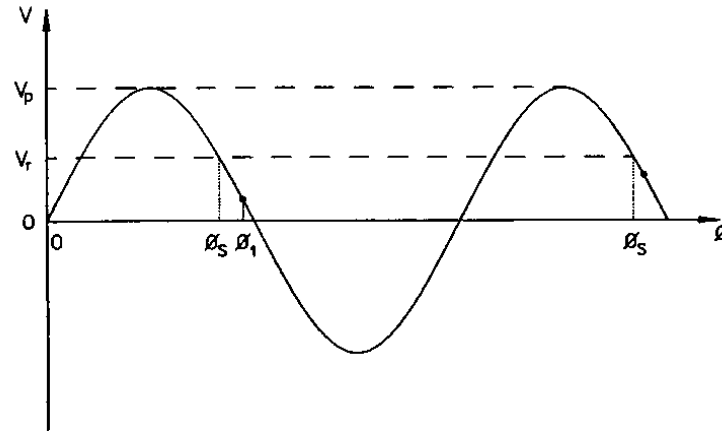
To replenish the energy loss by electrons as synchrotron radiation

How does it work ?

To provide a time varying electric field (voltage) that gives the electron a boost when it passes the accelerating gap (resonant cavity)



The bunching effect of the r.f. cavity



Cavity voltage varies sinusoidally with a phase angle ϕ . Since $(\phi/2\pi) \cdot \text{frequency} = \text{time}$, the ϕ axis is equivalent to time

ϕ_s (stable phase angle): the electron receives the correct amount of energy it lost to synchrotron radiation

If an electron passes through the gap late (ϕ_1), it will not receive sufficient energy and moves into an orbit of smaller radius and will take less time to arrive back at the gap. Thus electrons are said to be bunched by the r.f. cavity.

The potential well within which they are stable is called the **bucket**.

Time Structure

r.f. cavity bunches
the electrons. Bunch
length determines
the pulse width

Number of bunches
determines the
repetition rate

ALS, 1.9 GeV

328 buckets, bunch: 35 ps

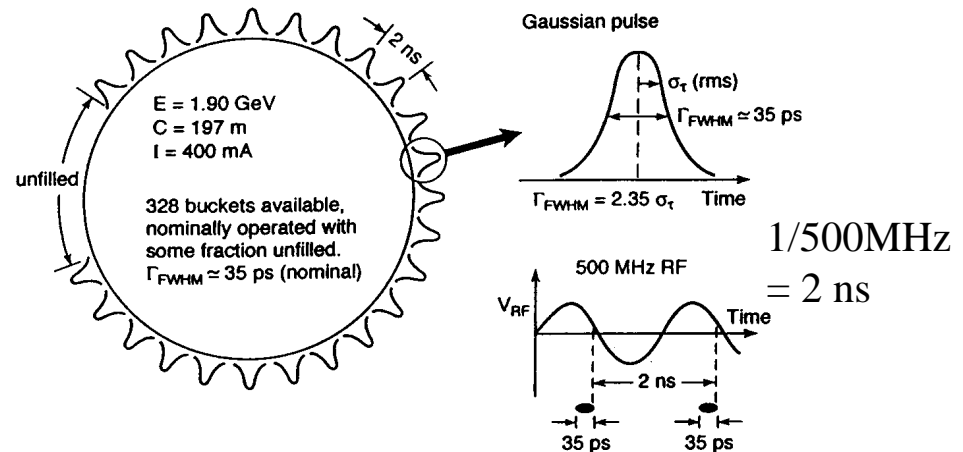
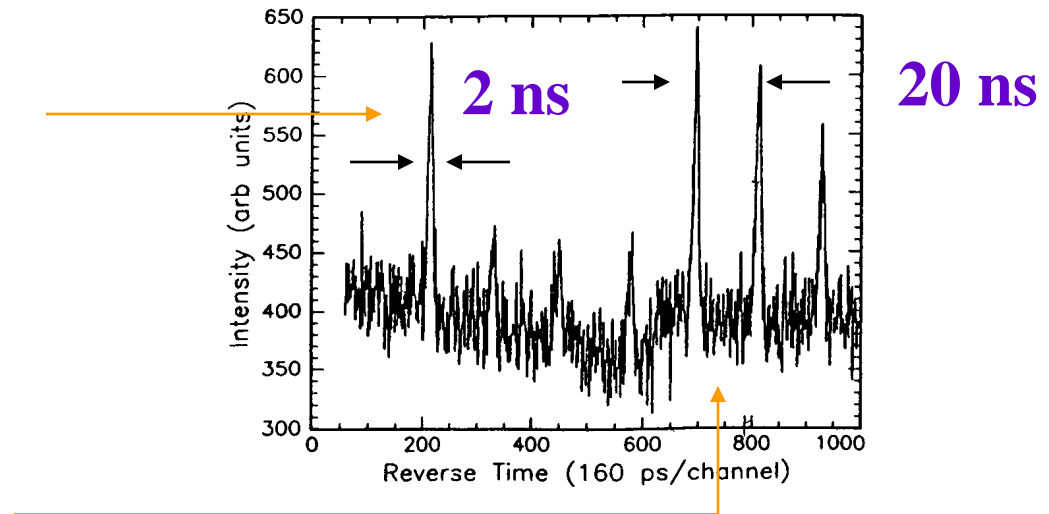
circumference = 197 m

$197/328 = 0.6$ m

Separating the bunches

$0.6\text{m}/3 \times 10^8 \text{ ms}^{-1} = 2 \text{ ns}$

NSLS UV



Relevant storage ring parameters (CLS)

Parameters	Main hardware	Relevance to users
Energy (e.g. 2.9GeV)	B field and radius $\lambda_c (\text{\AA}) = \frac{18.6}{E^2 B} = \frac{5.6\rho}{E^3}$	practical photon energy (λ_c)
Current (500 mA)	r.f. cavity: time varying voltage	photon flux
Emittance (16 nm-mrad)	magnetic lattice	Brightness (microbeam)
Circumference (8 straight sections)	r.f. cavity (500 MHz), ID	time structure (dynamics), microbeam

The magnetic system (lattices)

The system of magnetic optics that guides and focuses an electron beam is called the **lattice**. The choice of the lattice is the most critical decision for it determines

Emittance (electrons) – brightness (photon beam)

**Beam lifetime
and stability**

**No. of insertion
devices**

Cost

Magnetic Lattice

Lattice Characteristics

The cell: building block
arrangement of bending
magnet and focusing
magnets

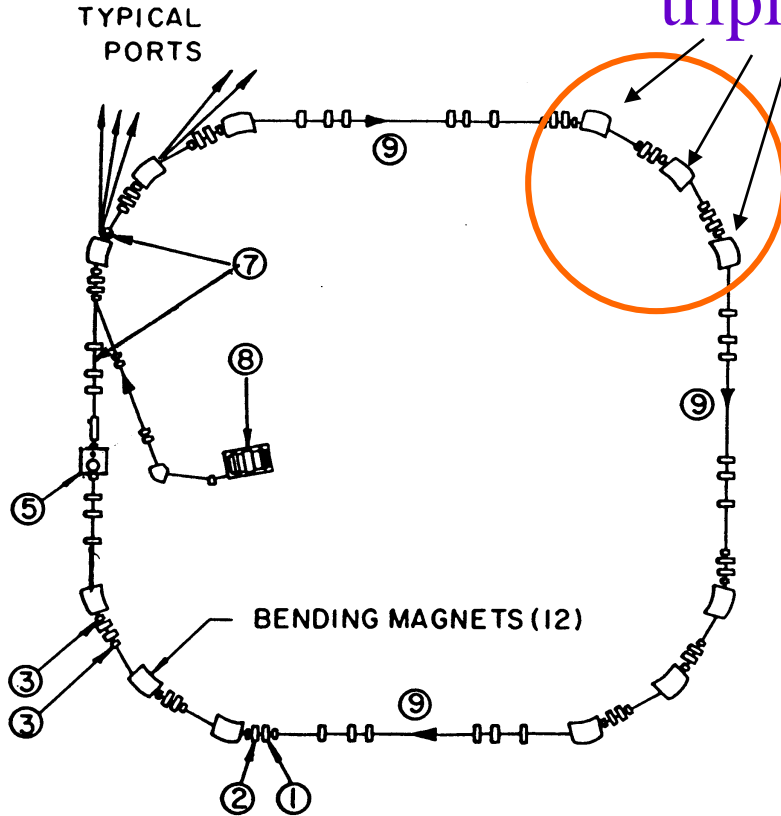
- a. Double bend Achromat
- b. Triple bend Achromat



New: Multiple bend achromat

The Aladdin Ring (Stoughton Wisconsin): Home of the Canadian Synchrotron Radiation Facility

triple bend



- 1 Focussing Quadrupole
- 2 Defocussing Quadrupole
- 3 Sextupole Correction
- 5 Accelerating Cavity
- 9 Straight Sections (available for Undulators and Wigglers)
- 7 Injection Kicker Magnets
- 8 100 MeV Microtron

**UV Ring NSLS,
Brookhaven national Lab**

The journey of the electron in the storage ring

Electron gun → Linac →

Booster

injection

Storage ring

BM → SR
and
ID

R.f cavity
replenishes
the energy
lost to SR

How a Synchrotron Works

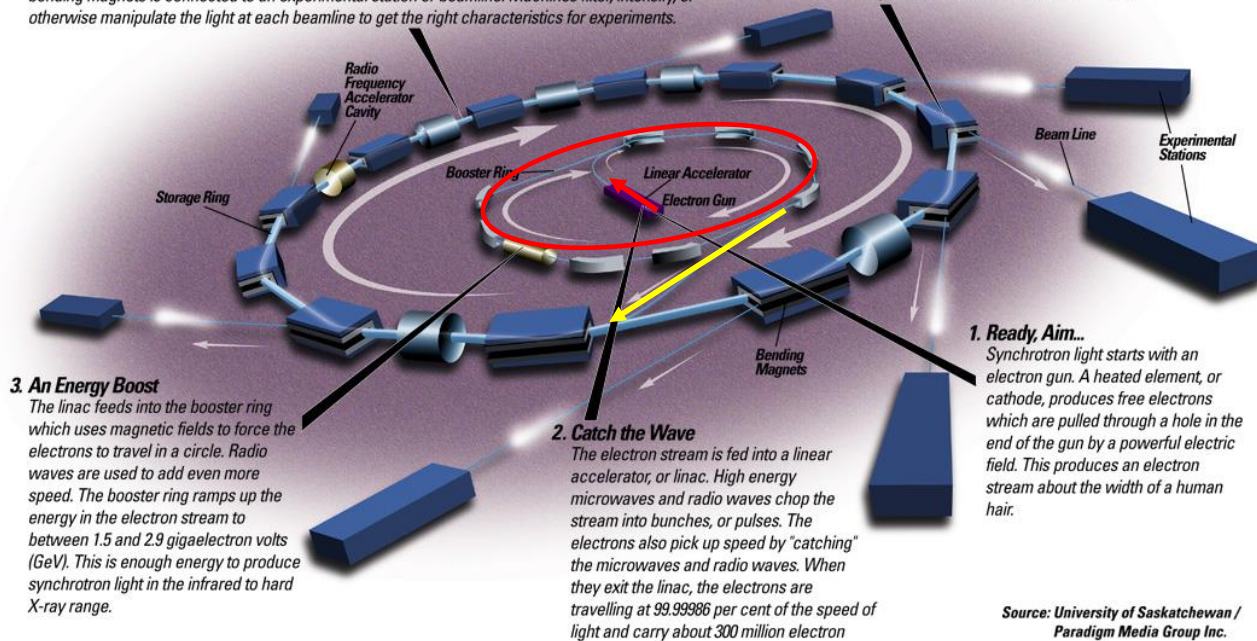
4. Storage Ring

The booster ring feeds electrons into the storage ring, a many-sided donut-shaped tube. The tube is maintained under vacuum, as free as possible of air or other stray atoms that could deflect the electron beam. Computer-controlled magnets keep the beam absolutely true.

Synchrotron light is produced when the bending magnets deflect the electron beam; each set of bending magnets is connected to an experimental station or beamline. Machines filter, intensify, or otherwise manipulate the light at each beamline to get the right characteristics for experiments.

5. Focusing the Beam

Keeping the electron beam absolutely true is vital when the material you're studying is measured in billionths of a metre. This precise control is accomplished with computer-controlled quadrupole (four pole) and sextupole (six pole) magnets. Small adjustments with these magnets act to focus the electron beam.



Source: University of Saskatchewan / Paradigm Media Group Inc.

Beamlines and experimental stations

Beamline optics solution:

general considerations

1. Energy region (monochromator)
2. Photon intensity vs. resolution
(slits and mirrors)
3. Spatial resolution (emittance, focusing)
4. Polarization (undulator)
5. Coherence (undulator)

Optical elements

a. Aperture/slits (resolution and flux trade-off)

b. Mirrors

(collimation, focus, higher order rejection)

a. Monochromators (monochromatic light)

IR- gratings, FTIR (interferometer)

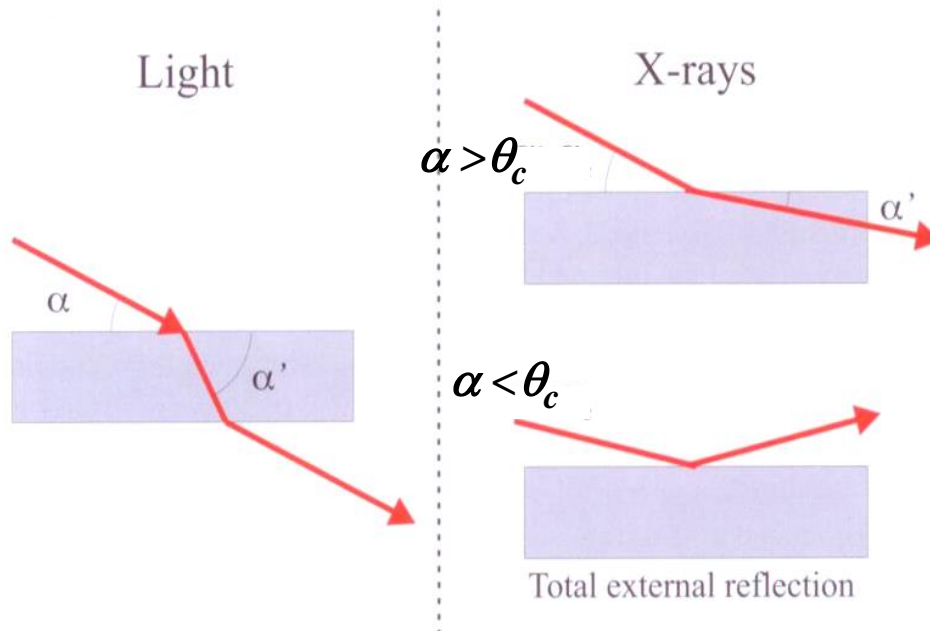
UV, VUV (3- 100 eV) – gratings

VUV- soft x-ray gratings (100 eV -5000 eV) and crystals with large $2d$ values, e.g. InSb(111)

Hard x-rays – crystals (Si(111), Si (220) etc.)

X-ray Mirrors

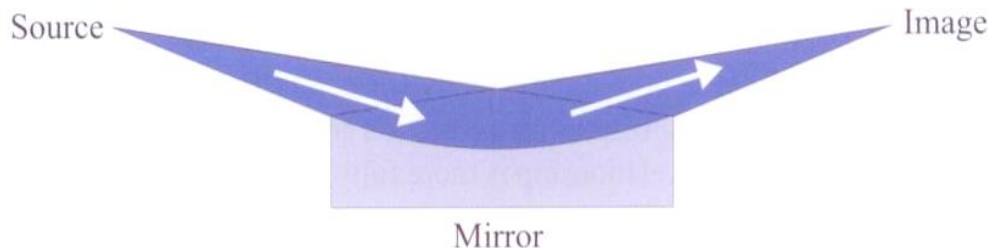
(a) Refraction and reflection of light and X-rays



The index of refraction for X-rays is slightly less than 1

θ_c : critical angle where total reflection occurs

(b) Focusing X-ray mirror



Plane mirror: collimation;
Curved mirror (spherical, elliptical, etc.): focusing

Mirror and X-ray reflectivity

Let θ_c be the angle at which total reflection occurs, then

$$\sin \theta_c = \lambda(Nr_0\pi)^{1/2}$$

N: # of electrons per cm³

at glancing angle

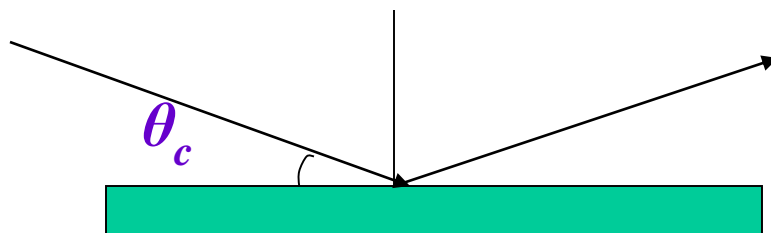
$$\theta_c \sim \sqrt{\delta} \sim (2.74 \times 10^{-6} Z \rho A)^{1/2} \lambda$$

$$n = 1 - \delta + i\beta$$

atomic #, density, atomic mass

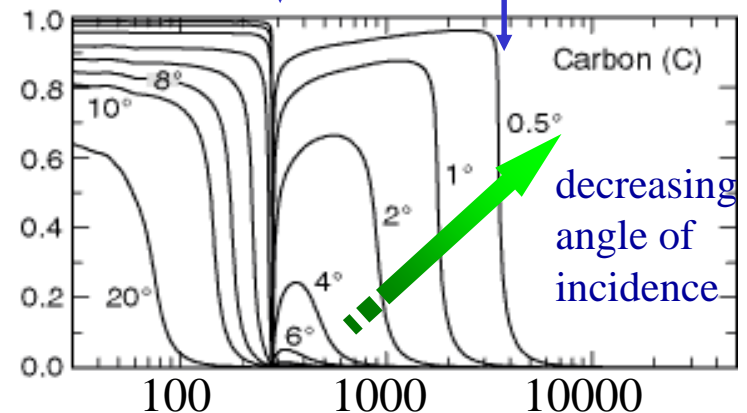
C K-edge @ 290eV

high energy photon cutoff



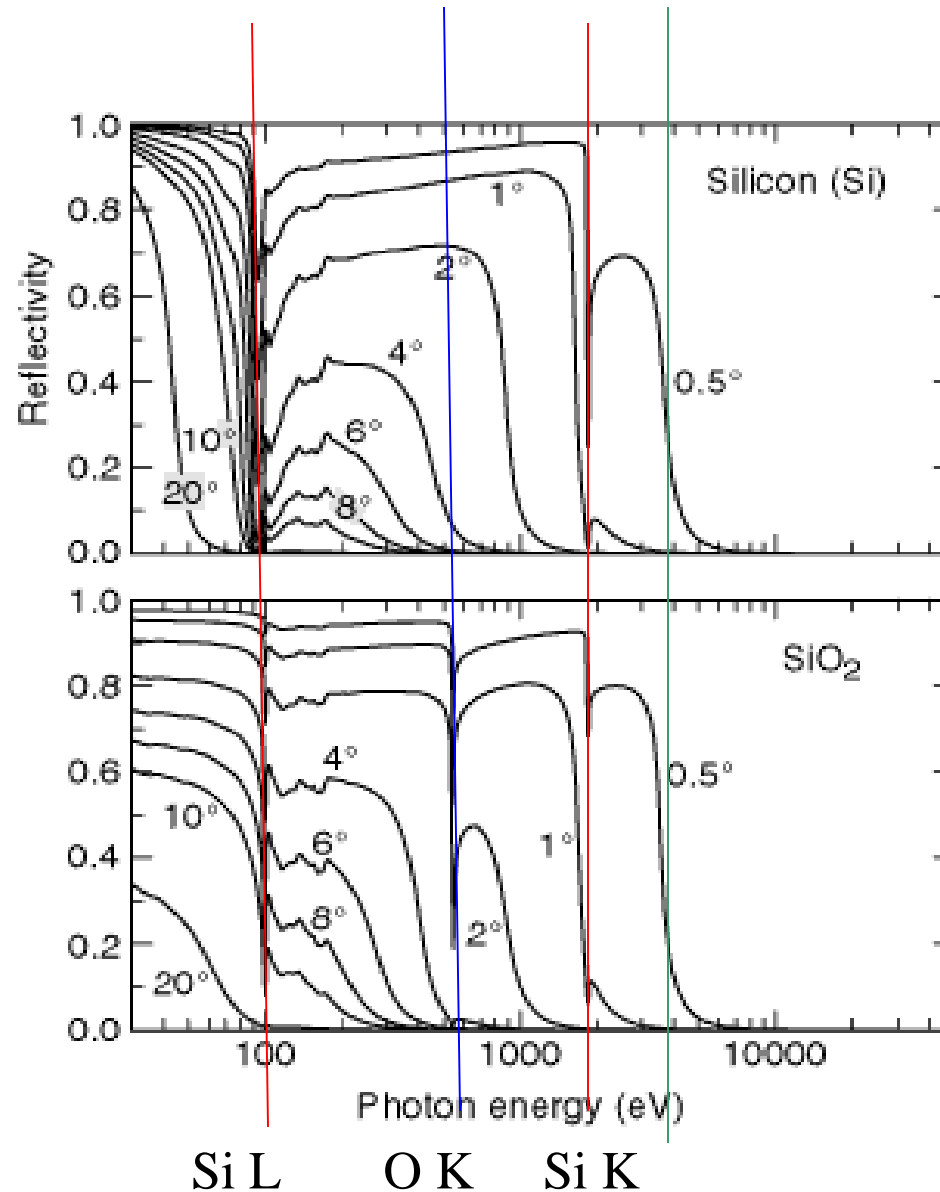
X-ray mirror

1 rad = 57.3 °, 1° = 17.45 mrad



Photon energy (eV)

Reflectivity as a function of incident energy and angle

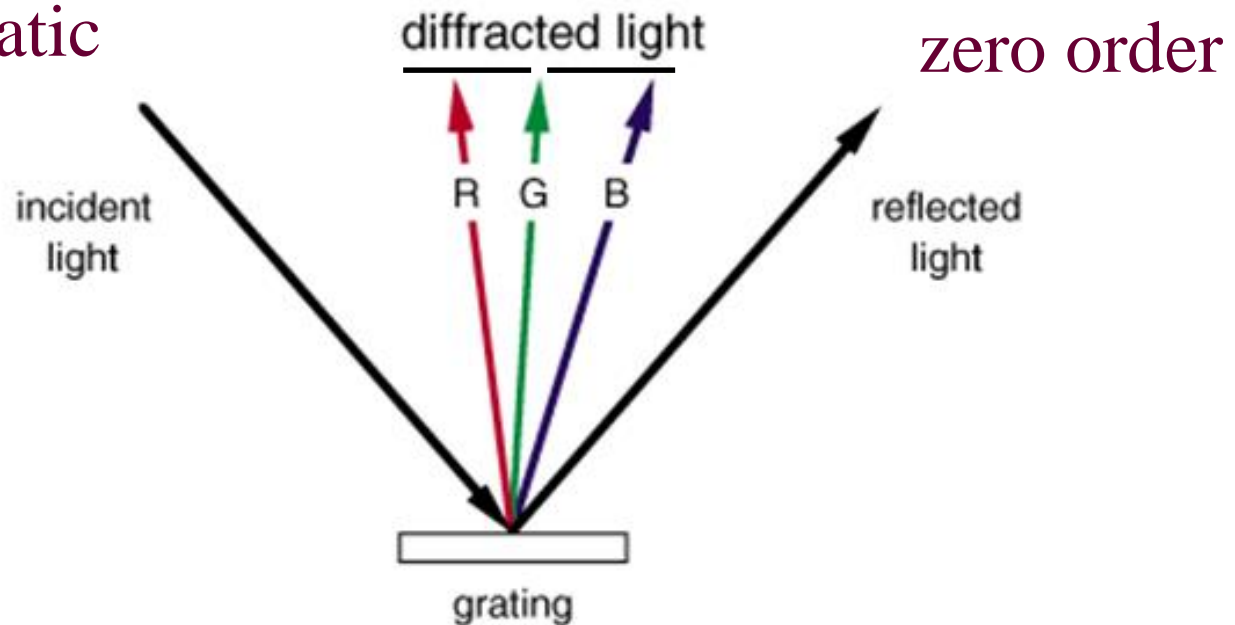


The **energy cutoff** at 0.5° angle of incidence is ~ 4 keV; this property can be used to filter out high energy photons (higher order)

What is a grating and a grating monochromator?

Monochromatic light can be selected with a moving aperture or by rotating the grating

polychromatic



Grating: arrays of lines with well defined separation and profile

Grating equation

Consider a plane grating, the grating equation is

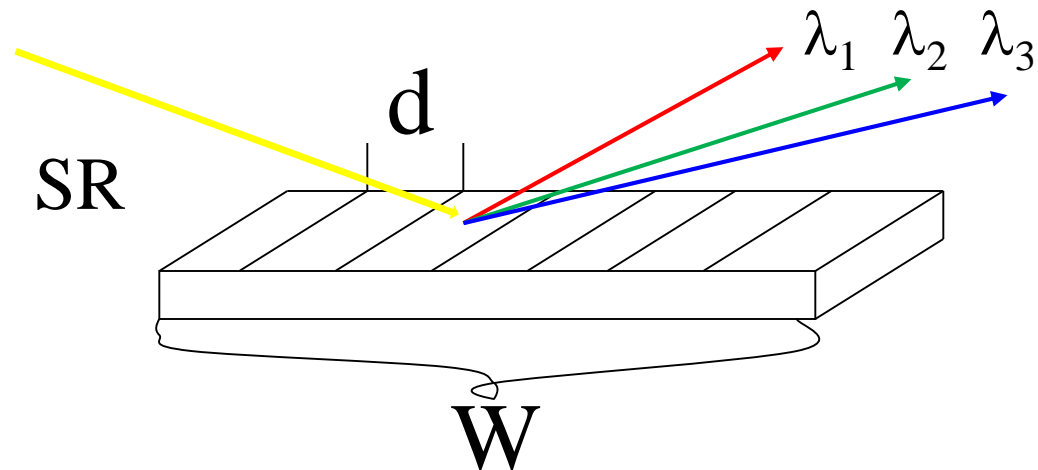
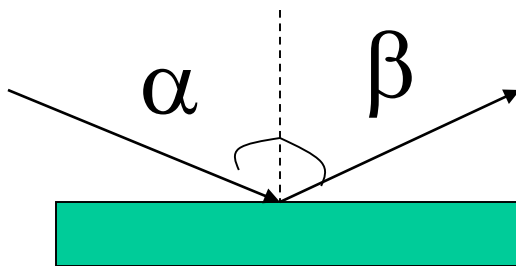
$$n\lambda = d(\sin\alpha + \sin\beta)$$

n : order of the diffraction; d : distance between the grating lines;
 $d = W/N$ where W is the ruled width and N the no. of lines.

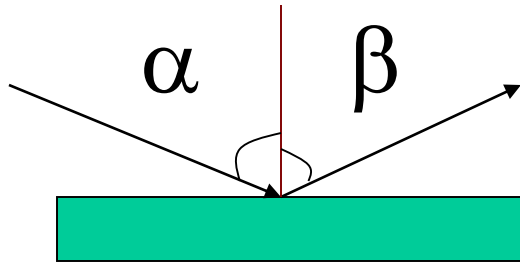
A 800 line grating means 800 lines per mm

α : angle of incidence β : angle of diffraction

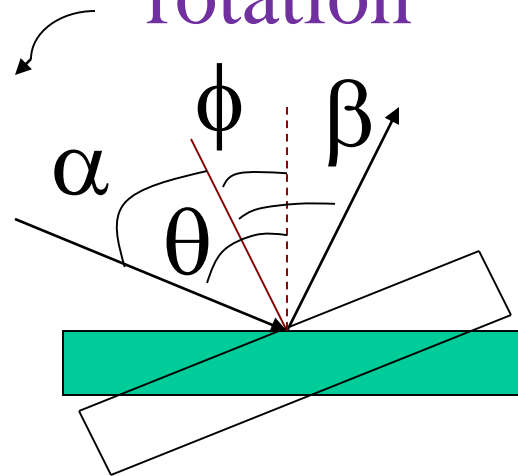
zero order position, $\alpha = -\beta$



zero order normal



rotation



Φ : angle of rotation of grating away from the zero order position

θ : angle between zero order normal and incident beam

$$\alpha = \theta - \phi, \quad \beta = -\theta + \phi; \quad n\lambda = 2d \cos\theta \sin\phi$$

The resolution: $E/\Delta E = \lambda/\Delta\lambda \propto N_l n$

It can be seen high resolution can be obtained with high line density (N_l) grating or the use higher order (n) radiation. E.g.: A 1800 line grating has better resolution than a 1200 line grating

Double Crystal Monochromator (DCM)

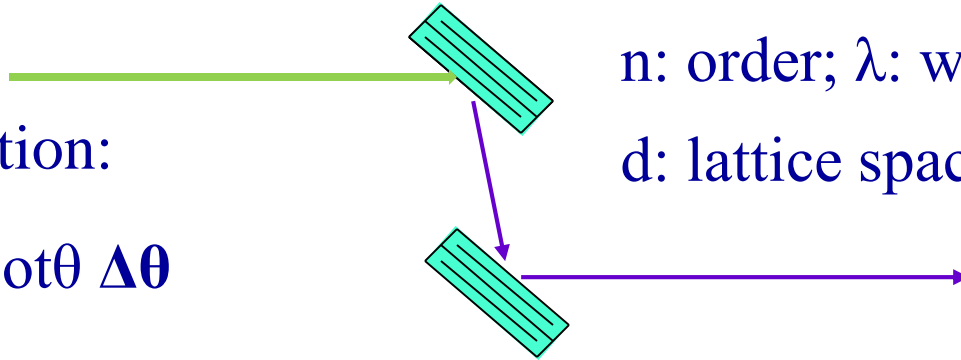
Bragg's law $n\lambda = 2d \sin\theta$

n: order; λ : wavelength;

d: lattice spacing; θ : Bragg angle

The resolution:

$$\Delta\lambda/\lambda = \cot\theta \Delta\theta$$



$\Delta\theta$ depends on the inherent width of the crystal (Darwin curve/**rocking curve**) and the vertical angular spread of the SR

$$\Delta\theta = \sqrt{\Delta\theta_{SR}^2 + \Delta\theta_C^2}$$

$$\Delta\theta_{SR} = \psi$$

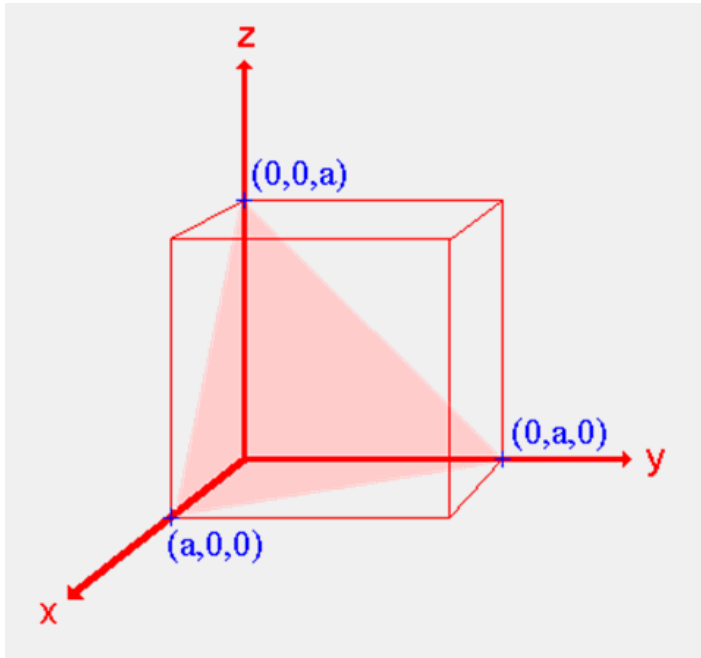
$$\sim (1/\gamma) 0.57 (\lambda/\lambda_C)^{0.43} \text{ (rad)}$$

For most crystals $\Delta\theta_C > \Delta\theta_{SR}$. Crystals used for DCM

Crystal	Bragg Reflection	2d(Å)
Si	(111)	6.271
InSb	(111)	7.481

Crystal planes and Miller indices

Miller indexes (h,k,l) are used to define parallel planes in a crystal from which the inter-planar spacing d can be obtained



For a cubic crystal, the Miller indices (111) refers to the set of planes that is parallel to the plane that intercepts the three axes at $x = a$, $y = a$ and $z = a$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

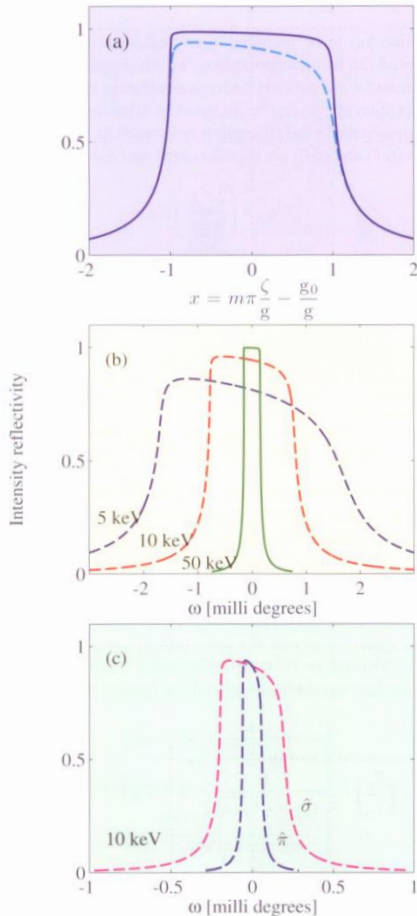
Exercise: work out the lattice spacing for Si(111) shown above

What is a Darwin (rocking) curve ?

(a) Darwin curve of Si(111), reflectivity is 100% for x between -1 and 1,

(b) Darwin curve of Si(111) as a function of rotation angle @ 3 energies

(c) Darwin curve of Si(333) with different polarization



$$W = \zeta \tan \theta$$

	$\Delta\theta_C = w = \zeta \tan \theta$ (mrad)		
	(111)	(220)	(400)
$h\nu = 8050 \text{ eV}$	$\tan \theta = 0.2534$	$\tan \theta = 0.4379$	
Si $a = 5.4309 \text{ \AA}$	$\zeta = 139.8 \times 10^{-3}$ mrad	$\zeta = 61.1.8 \times 10^{-3}$	$\zeta = 26.3 \times 10^{-3}$
Ge $a = 5.6578 \text{ \AA}$	$\zeta = 347.2 \times 10^{-3}$	$\zeta = 160.8 \times 10^{-3}$	$\zeta = 68.8 \times 10^{-3}$